$$N_{12} = \sum_{n_1}^{n_2} N_n \, \Delta n \simeq \int_{n_1}^{n_2} N_n \, dn \, , \tag{5.8}$$

where N_n is a large number and Δn is equal to 1.

Substitution of Eq. (5.7) into Eq. (5.8) and integration over the interval (n^*, n^∞) gives the number of clusters N^* that exceed the critical size for a sudden change in driving force:

$$N^* = N \int_{n^*}^{n^\infty} \exp\left(- \frac{\Delta W^\circ(n)}{kT_0}\right) dn \, , \tag{5.9}$$

where $\Delta W^\circ(n)$ is given by Eq. (5.4) for the initial stable state ahead of the shock front, and n^* is given by Eq. (5.5) for the state behind the shock front. We are not able to integrate Eq. (5.9) analytically. However, we can differentiate Eq. (5.9) with respect to G_{21} using Leibnitz's rule,[54] which results in the expression,

$$\begin{aligned} \frac{dN^*}{dG_{21}} &= N \exp\left(- \frac{\Delta W^\circ(n^\infty)}{kT_0}\right) \frac{dn^\infty}{dG_{21}} \\ &\quad - N \exp\left(- \frac{\Delta W^\circ(n^*)}{kT_0}\right) \frac{dn^*}{dG_{21}} \\ &\quad + N \int_{n^*}^{n^\infty} \frac{\partial}{\partial G_{21}} \exp\left(- \frac{\Delta W^\circ(n)}{kT_0}\right) dn \, . \end{aligned} \tag{5.10}$$

Since the integrand and n^∞ are independent of G_{21},

$$\frac{dN^*}{dG_{21}} = -N \exp\left(- \frac{\Delta W^\circ(n^*)}{kT_0}\right) \frac{dn^*}{dG_{21}} \, . \tag{5.11}$$

Differentiating Eq. (5.5) with respect to G_{21} results in the expression,

$$\frac{dn^*}{dG_{21}} = -n^* \left(\frac{3}{G_{21}} - \frac{2}{V_2} \frac{dV_2}{dG_{21}} \right) . \qquad (5.12)$$

Substituting Eq. (5.12) into Eq. (5.11) results in the expression,

$$\frac{dN^*}{dG_{21}} = Nn^* \left[\frac{3}{G_{21}} - \frac{2}{V_2} \frac{dV_2}{dG_{21}} \right] \exp\left(- \frac{\Delta W^\circ (n^*)}{kT} \right) . \qquad (5.13)$$

Eliminating n^* and $\Delta W^\circ (n^*)$ by Eqs. (5.4) and (5.5) results in the final expression,

$$\frac{dN^*}{dG_{21}} = \frac{-32\pi\sigma^3 V_2^2 N}{3G_{21}^3} \left(\frac{3}{G_{21}} - \frac{2}{V_2} \frac{dV_2}{dG_{21}} \right)$$
$$\times \exp\left[\frac{-16\pi\sigma^3 V_2^2}{G_{21}^2} \left(1 - \frac{2G_{21}(P=0, T=295^\circ K)}{3G_{21}} \right) \right] , \qquad (5.14)$$

which depends on σ, V_2, N, dV_2/dG_{21}, and G_{21}. This equation is significant because it establishes a relation between number of nucleation sites and driving force, G_{21}, in the stable field of phase 2.

To calculate values of dN^*/dG_{21}, from Eq. (5.14) requires values for surface energy σ, volume V_2, N, driving force G_{21}, and dV_2/dG_{21}. Values for σ found in the literature vary from 20 ergs/cm^2 for a coherent twin interface to 200 ergs/cm^2 for an incoherent interface.[51,55] Examination of Eq. (5.14) reveals that the influence of the exponential function is overriding and if any nucleation is to occur, the argument of